Predicting Road Traffic Flows for Urban Transportation Planning*

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Abstract

The standard method for predicting traffic flows on urban road networks, called static traffic assignment, is based on the principle that drivers seek their own least cost routes from their origins to their destinations. This principle corresponds to a network user equilibrium in which all used routes have equal costs and no unused route has a lower cost, for every origin-destination pair. Under somewhat mild assumptions, the network equilibrium problem can be formulated as a convex optimization problem with linear constraints, and solved with an iterative algorithm. The precision and speed of such solutions has increased remarkably during the past ten years.

Although the total flows on links of the urban road network are uniquely determined in this formulation, route flows and multiple-class link flows are not. An additional assumption, called the condition of proportionality, is required to determine these flows uniquely. This assumption is the basis for solving these problems in a new algorithm, Traffic Assignment by Paired Alternative Segments (TAPAS).

In this paper, the findings of computational experiments pertaining to three network representations and three trip matrices are presented. The network representations pertain to restrictions on the use of certain links of the network by trucks in a multiple-class assignment. The trip matrices represent a range of sensitivities of travelers to generalized costs. The findings are presented in a way that transportation planning professionals may find helpful in understanding the importance of network representations, multiple-class assignments and the condition of proportionality to their travel forecasting practice.

Key words: user-equilibrium traffic assignment; route flows; multiple-class link flows; proportionality condition

* Based on joint research with Hillel Bar-Gera, Ben-Gurion University of the Negev, Beer-Sheva, Israel, and Yu (Marco) Nie, Northwestern University, Evanston, Illinois, USA.
1. Introduction

The standard method for predicting traffic flows on urban road networks, called static traffic assignment, is based on the principle that drivers seek their own least cost routes from their origins to their destinations. This principle corresponds to a user equilibrium (UE) in which all used routes have equal costs and no unused route has a lower cost, for every origin-destination pair (Beckmann et al., 1956). Under somewhat mild assumptions, the problem can be formulated as a convex optimization problem with linear constraints, and solved with an iterative algorithm.

Solution methods for the standard traffic assignment problem, in which link travel times/costs depend only on their own flows, have advanced steadily over the past 60 years. Initially, solution methods were heuristic, and not known to converge to the equilibrium solution. In fact, until the mid-1970s most practitioners were not aware of Martin Beckmann’s formulation, and simply sought a solution that corresponded to their intuition about cost-minimizing route choice behavior. Examples of such heuristic methods are the iterative capacity-restraint method implemented at the US Department of Commerce (1964) in the 1950s and the incremental method implemented by the Control Data Corporation (1965) in TRAN/PLAN.

Convergent assignment methods based on Beckmann’s formulation began to emerge in the late 1960s and early 1970s in early Ph.D. research in operations research and regional science (Dafermos and Sparrow, 1969; Bruynooghe et al, 1969; Nguyen, 1974; LeBlanc et al., 1975). The method that became most widely used was based on the proposal of Frank and Wolfe (1956), ironically published in the same year as Beckmann’s original formulation. One of the originators of this linearization algorithm was Larry LeBlanc; another was Suzanne Evans (1976), who extended the formulation to include origin-destination choice (trip distribution), and extensively studied the mathematical properties of a partial linearization algorithm. Their algorithms solved the assignment problem in terms of link flows, and therefore are termed link-based. The link-based method rather quickly replaced the older heuristic methods during the 1980s, when software systems based on mini- and micro-computers began to be offered. Most commercial travel forecasting software systems now include a link-based solution method. A route-based method was proposed by Bothner and Lutter (1982), and applied in a software system developed by PTV AG based in Karlsruhe, Germany. This method is also known to converge to the user-equilibrium solution.

Refinement of these solution methods continued through the 1990s, but few innovations occurred until the origin-based assignment (OBA) method was proposed by Hillel Bar-Gera (2002). Bar-Gera’s algorithm included several innovations; an important one was the organization of the search with an origin-based subnetwork defined for each origin zone, earlier called a ‘bush’ by Robert Dial (1971). Bar-Gera’s algorithm was able to achieve precise solutions of the assignment problem for the first time, although the solution times were relatively long. Later, Dial (2006) proposed a bush-based algorithm, which solved the assignment problem precisely and more quickly. The precision of the solution actually needed for practice depends on its use. One of the main applications of travel forecasts is to compare scenarios. Such comparisons only make sense if the precision of each solution is substantially better than the differences among the scenarios. Boyce et al. (2004) explored the effects of solution precision on such scenario differences, recommending a Relative Gap no greater than 1E-4.
A property of Beckmann’s formulation, and hence of all related solution algorithms, is that route flows are not uniquely determined. A related non-uniqueness property pertains to multiple-class link flows, in the event that two or more trip matrices are assigned to the network. The basic reason is that link travel times are functions of total link flows, and not of route or class flows. Therefore, route or class flows can be swapped among routes or classes, leaving the total user-equilibrium link flows unchanged. In order to determine uniquely the route flows, or multiple-class link flows, an additional assumption is needed, called the ‘condition of proportionality.’ The condition of proportionality states that the same proportions apply to all travelers facing a choice between a pair of alternative segments (PAS), regardless of their origins and destinations; a segment is defined as a sequence of one or more links. The main reasons to adopt proportionality are: (1) it is a reasonable condition that is easy to understand; (2) it offers consistent treatment, which may be important if equity issues are present; (3) it results in stable solutions with respect to model inputs (Lu and Nie, 2010); and (4) satisfaction of the condition of proportionality can be tested directly for any solution (Bar-Gera et al., 2010). These properties contribute to a solution’s usefulness, especially as the only alternative presently is to consider any choice among all equilibrium solutions as acceptable, even if it is random or arbitrary.

An important implication of proportionality is that any route that can be used under the UE condition should be used. Therefore, a way to satisfy proportionality is to make sure that ‘no minimum cost route is left behind.’ This condition is particularly important in the design of algorithms that aim to satisfy proportionality. It is also helpful for the interpretation of the results found in Sections 3 and 4. Traffic Assignment by Paired Alternative Segments (TAPAS) is a UE assignment algorithm designed to compute route flow and multiple-class link flow solutions that satisfy the condition of proportionality (Bar-Gera, 2010). TAPAS was used to prepare the assignments presented in this paper; therefore, the condition of proportionality is satisfied in these results. Assignments with TAPAS are termed PAS-based.

During the past three years, experiments were conducted with TAPAS as well as link-based, route-based and bush-based algorithms. Five commercially available assignment tools were applied in seven case studies (Boyce et al., 2010). Subsequently, using the data from the Chicago region case study, more experiments were performed, which form the basis for this paper. The earlier Chicago region experiments consisted of the assignment of a single-class trip matrix consisting of the sum of car and truck matrices. In these assignments, no restrictions were placed on links used by trucks. Then, a two-class assignment was performed in which trucks were restricted from car-only lanes on two major expressways and the Lake Shore Drive. These assignments are termed ‘partial truck restrictions.’ However, trucks are actually restricted from operating on many additional arterial streets in the City of Chicago and in some nearby suburbs. These additional truck restrictions, called ‘full truck restrictions’ were coded into a network to form the basis for evaluating the effect of these restrictions on the overall assignment results.

The remainder of the paper presents figures and commentary on experiments performed with TAPAS. Section 2 describes in more detail the properties of the trip matrices and networks. Section 3 examines the effect of partial and full restrictions on truck use on regional link flows, and on four selected segments. Section 4 considers the effect of different car trip matrices on overall link flows and two of the selected segments. Section 5 briefly discusses some of the implications of the findings for transportation planning practice.
2. Details of the Trip Matrices and Their Assignment to the Road Networks

The experiments presented in this paper are based on the assignment of three car trip matrices computed for the Chicago region for 1990 conditions with a combined model of origin-destination-mode choice for the morning peak period (6:30 – 8:30 am), given a fixed truck trip matrix. The Chicago regional zone system for 1990 consisted of 1790 zones (Map 1a); the road network had 12,982 nodes and 39,018 links (Map 1b). Total regional person trips by car and transit were estimated to be 1.513 million persons per hour, whereas the total truck trips were 0.455 million passenger-car-equivalents per hour, both representative of the morning peak period. The person trips were based on exogenous estimates of the total numbers of persons departing from and arriving at each zone by car and transit during the two-hour peak period.

As described in more detail in the Appendix, origin-destination flows of persons by mode were computed with a doubly-constrained, negative exponential (logit) function. The generalized travel costs on which the flows were based are the endogenous user-equilibrium times and travel distances consistent with the model-determined car flows, an exogenous truck trip matrix, and fixed travel times and fares for transit. A key parameter in the logit function is the sensitivity of travelers to generalized costs. A large value of this parameter means that travelers are more sensitive to travel times, distances and fares, whereas a small value means they are less sensitive. For small values, trips lengthen in time and distance, increasing congestion over the road network; therefore, more travelers choose transit, as compared with larger cost sensitivity values.

In this research, three cost sensitivity (CS) values were studied: 0.20, 0.10 and 0.05. The results presented in Figures 1-6 pertain to the first value, which is considered to be somewhat realistic; the mean travel time for this value is 20 minutes for interzonal car travel, with a corresponding transit travel time of 27 minutes. Following the presentation of findings for this parameter value, comparisons are made with the two smaller values in Figures 7-9. Table 1 shows selected summary measures for the three CS values for interzonal and intrazonal travel, by mode, and also for trucks. Examination of these totals and means should enable one to grasp the overall characteristics of the trip matrices.

In initial experiments with these trip matrices, the car and truck matrices were summed to form a single-class matrix. In many urban regions, however, trucks are restricted from using certain roads. These restrictions include car-only lanes of expressways, scenic parkways and boulevards. In the Chicago region specifically, two major expressways have car-only express lanes from which trucks are prohibited. In addition, trucks cannot use Lake Shore Drive, a 24 km multi-lane, grade-separated roadway along Lake Michigan. Using these truck restrictions, which are called ‘partial truck restrictions,’ two-class assignments were prepared. Subsequently, a network was coded in which trucks were restricted from the extensive boulevard system in the City of Chicago, partially illustrated in Map 1c, and from other roadways in residential neighborhoods to the north and west of the city. The Rand-McNally Motor Carriers’ Atlas for 1991 (Map 1d) was consulted for additional truck restrictions (Rand-McNally, 1991, p. 23). Altogether, trucks were restricted from over 500 links of the road network, which is called ‘full truck restrictions.’ All assignments were computed to a Relative Gap less than 1E-10, as defined in the Appendix.
Map 1. Chicago Regional Zone System and Road Networks

1a. Chicago regional zone system

1b. Chicago regional road network

1c. City of Chicago boulevard system (partial)

1d. Rand-McNally Motor Carriers’ Road Atlas
The inclusion of these truck restrictions may be expected to improve substantially the validity of the traffic assignments. Not only are trucks removed from these car-only roadways, but also more capacity is available for use by cars, as compared with the single-class case. One objective of these studies, then, is to determine the effect of these partial and full truck restrictions on the assignment results. A second objective is to analyze the effects of the three values of the cost sensitivity parameter on the assignments. A third objective is to observe the effect of the condition of proportionality on the flows on individual links.

Table 1. Summary Measures for the Chicago Region for the Morning Peak Period, 6:30-8:30 am, 1990

<table>
<thead>
<tr>
<th>Zone Pairs</th>
<th>Cost Sensitivity</th>
<th>Persons</th>
<th>Vehicles</th>
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<tr>
<td></td>
<td>car</td>
<td>transit</td>
<td>transit (%)</td>
</tr>
<tr>
<td>Flow</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(___/hour)</td>
<td>0.05</td>
<td>903,191</td>
<td>590,691</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>1,021,740</td>
<td>456,588</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>1,116,689</td>
<td>331,028</td>
</tr>
<tr>
<td>Total travel time</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(hours)</td>
<td>0.05</td>
<td>623,863</td>
<td>434,891</td>
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<tr>
<td></td>
<td>0.10</td>
<td>473,016</td>
<td>256,196</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>363,314</td>
<td>147,549</td>
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<tr>
<td>Mean travel time</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(minutes)</td>
<td>0.05</td>
<td>41.4</td>
<td>44.2</td>
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<tr>
<td></td>
<td>0.10</td>
<td>27.8</td>
<td>33.7</td>
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<tr>
<td></td>
<td>0.20</td>
<td>19.5</td>
<td>26.7</td>
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<table>
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<th>transit (%)</th>
<th>car</th>
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<td>Flow</td>
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<td>(___/hour)</td>
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<td>996</td>
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<td>64,971</td>
<td>523</td>
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<td>54,142</td>
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<td>Total travel time</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(hours)</td>
<td>0.05</td>
<td>5,145</td>
<td>612</td>
<td>10.6</td>
<td>4,288</td>
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<td>486</td>
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<td>0.20</td>
<td>13,895</td>
<td>214</td>
<td>1.5</td>
<td>11,580</td>
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<tr>
<td>Mean travel time</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(minutes)</td>
<td>0.05</td>
<td>16.9</td>
<td>34.0</td>
<td>16.9</td>
<td>13.8</td>
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<td>15.0</td>
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<tr>
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<td>0.20</td>
<td>12.8</td>
<td>24.5</td>
<td>12.8</td>
<td>13.8</td>
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3. Effect of No, Partial and Full Truck Restrictions on Road Traffic Assignments

Assignment results showing the effect of truck restrictions for the cost sensitivity parameter value equal to 0.20 are presented in Figures 1-6. Each figure consists of four parts:

1. comparison of single-class (SC) total flows with no truck restrictions (y-axis) with multiple-class (MC) total flows with full truck restrictions (x-axis);

2. comparison of multiple-class total flows with partial truck restrictions (y-axis) with multiple-class total flows with full truck restrictions (x-axis);

3. comparison of car flows with partial truck restrictions (y-axis) with car flows with full truck restrictions (x-axis);

4. comparison of truck flows with partial truck restrictions (y-axis) with truck flows with full truck restrictions (x-axis);
Figure 1 shows link flows; Figures 2-6 show origin-destination (OD) flows. The four parts of each figure are designed to enable the reader to assess the effects of the network representation and the use of multiple-class assignments on the results.

Figures 1.1 and 1.2 show the total link flows, ranging from 0 to 14,000 vehicles per hour (vph). In Figure 1.1, substantial differences in link flows are observed, resulting from the imposition of full truck restrictions. In the MC solution, flows on links lying above the 45-degree line are lower, while flows on links lying below the 45-degree line are higher, as compared with the SC solution. Presumably, the former are truck-restricted links. Likewise, in Figure 1.2, substantial differences in arterial link flows are observed between 0 and 3,000 vph, resulting from the restriction of trucks from boulevards and other streets. Some differences in flow on higher flow links may be also noted above 6,000 vph. Figures 1.3 and 1.4 break down total flows into their car and truck components. Note the difference in scale of these two figures. Higher car flows on arterial links with flows between 0 and 2,500 vph are clearly shown in Figure 1.3. Likewise, Figure 1.4 shows both higher and lower truck flows between 0 and 3,000 vph, and both lower and higher truck flows between 5,000 and 12,000 vph on links with full truck restrictions. Truck flows greater than 9,000 vph pertain to expressway links. The ‘root mean square error’ shown on the figures actually is the root mean square ‘difference’ of the arrays.

Figures 2-5 show OD flows using two selected links, North Avenue eastbound (EB), located two miles north of the Central Business District (CBD), and Harlem Avenue southbound (SB), located 8 miles west of the CBD. North Avenue is a four-lane arterial with a capacity of 1,540 vph in each direction, and Harlem Avenue is a four-lane arterial with a capacity of 1,980 vph in each direction, where capacity is the maximum flow at level of service D, formerly called ‘practical capacity’ and not a strict limit. Both links may be regarded as typical arterials on the one-mile street grid of Chicago and its inner suburbs. Neither arterial has truck restrictions.

Figures 2.1 and 2.2 show lower total OD flows and numbers of OD pairs using North Avenue EB with the truck restrictions imposed, as shown at the top of each figure and on the axes. Some larger SC flows for several OD pairs are smaller in the MC solution, as shown in Figures 2.1 and 2.2, while other OD flows are not affected. Figures 2.3 and 2.4 separate the effect of the additional truck restrictions on boulevards into cars and trucks; note the change in scale for truck flows. These figures show a slightly lower total link flow for cars, and slightly larger numbers of OD pairs for full truck restrictions. Four OD pairs for cars are substantially smaller in response to the truck restrictions on boulevards. Truck flows on a dozen OD pairs fall to near zero, and several OD pairs are added, as shown along the horizontal axis, for the full truck restrictions.

Figure 3 shows exactly the same points as in Figure 2 plotted on the log scale instead of the linear scale. The values shown on the axes are the orders of magnitude of the flow from 1E-4 to 1E+2. Zero flows are shown as 1E-4. A value of 1E0 corresponds to a flow of 1.0 vph. The impression of the pattern of flow in Figure 3 is dramatically different from Figure 2. In Figure 3.1, many small SC OD flows between 1E-4 and 1E0 are not found in the MC solution. Likewise, many small OD flows appear in the MC solution that were not found in the SC solution. As shown by Figure 3.2, these added flows are a result of the full truck restrictions; moreover, as seen in Figure 3.3, they are primarily car flows. Large numbers of OD flows, however, remain unchanged, as shown by the points along the diagonal line. From an
examination of Figures 2 and 3, we conclude that plots on both the linear and log scales are needed to visualize the full extent of differences among the three solutions for North Avenue EB.

Figures 4 and 5 show comparable results for Harlem Avenue SB, a suburban link somewhat west of some of the principal truck restrictions on arterial streets on the west side of Chicago. Although flows greater than 1 vph exhibit little change in the three solutions shown, substantial changes do occur with regard to flows less than 1 vph. The number of OD pairs in the MC solution is substantially than in the SC solution, although the link flow increases slightly.

Analyses of a pair of alternative segments (PAS) represent another way to compare the three assignments. According to the condition of proportionality, the OD flows over a pair of alternative segments having equal travel times should occur in the same proportion for every OD pair. In Map 2, an example of such a PAS is shown for North Avenue for node pair (8032, 10344). Segment 1 uses the same North Avenue link (6380, 6389) as in the select link analysis of Figure 2. Segment 2 uses a sequence of links somewhat to the north of that link.

Map 2. North Avenue Pair of Alternative Segments

Figure 6 shows the flows on Segment 2 (y-axis) as compared with Segment 1 (x-axis). The MC flows with full truck restrictions are shown as squares, while the SC flows with no truck restrictions and the MC flows with partial restrictions are shown as triangles. All OD flows form a straight line, demonstrating that proportionality is observed in each solution. The slopes of the lines are slightly different, suggesting only a modest change in the proportions among the three conditions. The pair of alternative segments has equal travel times for all three solutions, although the total flows vary slightly.

Summarizing these results for Figures 1-6, we conclude that the truck restrictions on arterial streets do cause subtle differences in the OD flows for two selected links, North Avenue and
Fig. 1. Comparison of solutions for links with no, partial and full restrictions on truck use for single-class (SC) and multiple-class (MC) PAS-based traffic assignments of car and truck matrices

Fig. 1.1 SC total link flows with no truck restrictions vs. MC total link flows with full truck restrictions

Fig. 1.2 MC total link flows with partial truck restrictions vs. MC total link flows with full truck restrictions

Fig. 1.3 MC car link flows with partial truck restrictions vs. MC car link flows with full truck restrictions

Fig. 1.4 MC truck link flows with partial truck restrictions vs. MC truck link flows with full truck restrictions
Fig. 2. North Avenue Select Link Analyses: Solutions with no, partial and full restrictions on truck use for single-class (SC) and multiple-class (MC) PAS-based traffic assignments (linear scale)

Fig. 2.1 SC total OD flows with no truck restrictions vs. MC total OD flows with full truck restrictions

Fig. 2.2 MC total OD flows with partial truck restrictions vs. MC total OD flows with full truck restrictions

Fig. 2.3 MC car OD flows with partial truck restrictions vs. MC car OD flows with full truck restrictions

Fig. 2.4 MC truck OD flows with partial truck restrictions vs. MC truck OD flows with full truck restrictions
Fig. 3. North Avenue Select Link Analyses: Solutions with no, partial and full restrictions on truck use for single-class (SC) and multiple-class (MC) PAS-based traffic assignments (log scale)

Fig. 3.1  SC total OD flows with no truck restrictions vs. MC total OD flows with full truck restrictions

Fig. 3.2  MC total OD flows with partial truck restrictions vs. MC total OD flows with full truck restrictions

Fig. 3.3  MC car OD flows with partial truck restrictions vs. MC car OD flows with full truck restrictions

Fig. 3.4  MC truck OD flows with partial truck restrictions vs. MC truck OD flows with full truck restrictions
Fig. 4. Harlem Avenue Select Link Analyses: Solutions with no, partial and full restrictions on truck use for single-class (SC) and multiple-class (MC) PAS-based traffic assignments (linear scale)

Fig. 4.1 SC total OD flows with no truck restrictions vs. MC total OD flows with full truck restrictions

Fig. 4.2 MC total OD flows with partial truck restrictions vs. MC total OD flows with full truck restrictions

Fig. 4.3 MC car OD flows with partial truck restrictions vs. MC car OD flows with full truck restrictions

Fig. 4.4 MC truck OD flows with partial truck restrictions vs. MC truck OD flows with full truck restrictions
Fig. 5. Harlem Avenue Select Link Analyses: Solutions with no, partial and full restrictions on truck use for single-class (SC) and multiple-class (MC) PAS-based traffic assignments (log scale)

Fig. 5.1 SC total OD flows with no truck restrictions vs. MC total OD flows with full truck restrictions

Fig. 5.2 MC total OD flows with partial truck restrictions vs. MC total OD flows with full truck restrictions

Fig. 5.3 MC car OD flows with partial truck restrictions vs. MC car OD flows with full truck restrictions

Fig. 5.4 MC truck OD flows with partial truck restrictions vs. MC truck OD flows with full truck restrictions
Fig. 6. North Avenue Analyses of a Pair of Alternative Segments: Solutions with no, partial and full restrictions on truck use for single-class (SC) and multiple-class (MC) PAS-based assignments

Fig. 6.1 SC total OD flows with no truck restrictions vs. MC total OD flows with full truck restrictions

Fig. 6.2 MC total OD flows with partial truck restrictions vs. MC total OD flows with full truck restrictions

Fig. 6.3 MC car OD flows with partial truck restrictions vs. MC car OD flows with full truck restrictions

Fig. 6.4 MC truck OD flows with partial truck restrictions vs. MC truck OD flows with full truck restrictions
Harlem Avenue, even though they are not near links on which trucks are restricted. These links were chosen for analysis because they had been used previously to compare various software systems and algorithms. Here, their use exhibits the stability of these precise solutions, with relatively marginal changes in major flows, but very substantial variations in minor flows.

4. Effect of Cost Sensitivity Values on Road Traffic Assignments

Figures 7-9 compare the effect of the cost sensitivity (CS) value on the single-class (SC) assignments versus the multiple-class (MC) assignments with full truck restrictions. The layouts of these three figures are similar. Each column shows the results for CS values of 0.20, 0.10 and 0.05, ranging from top to bottom.

The left-hand column of Figure 7 compares the total link flows for the SC flows (y-axis) with the MC flows (x-axis) for the three CS values. Note that Figure 7.1 is identical to Figure 1.1, and that the axes of the three figures have the same range to facilitate their comparison. Figures 7.2 and 7.3 show somewhat fewer differences between the SC and MC assignments as the CS values decrease, and congestion increases, as shown by the values of the total travel time of each assignment on each axis. The maximum flow on a network link increases to about 18,000 vph in Figure 7.3, as compared with less than 15,000 vph in Figure 7.1.

The right-hand column compares the link travel times for the SC and MC solutions for each CS value. Link travel times do not vary among the SC and MC assignments as much as link flows. However, the maximum link travel time in Figure 7.6 for CS = 0.05 is nearly 30 minutes, as compared with 12 minutes for CS = 0.20, suggesting the former solution is highly congested.

Figures 8 and 9 present select link analyses for North Avenue EB and Harlem Avenue SB for the three CS values. Linear scale plots are shown in the left-hand column, and log scale plots in the right-hand column. The following figures are repeated to facilitate comparison of the three CS values: 2.1 and 8.1; 3.1 and 8.4; 4.1 and 9.1; 5.1 and 9.4.

The following observations relate to the North Avenue EB plots:
1. The total link flow is similar for the SC and MC assignments, and slightly greater than the link’s capacity;
2. The total number of OD pairs is similar for CS values of 0.10 and 0.20, but greater for 0.05;
3. The maximum OD flow decreases from about 45 for CS = 0.20 to about 17 for CS = 0.05, suggesting the OD pairs are dispersed over more user-equilibrium routes in the 0.05 solution;
4. The number of OD pairs lying above the 45 degree line in all three plots suggests that the larger SC flows are smaller in the MC solution;
5. The log plots reveal that large numbers of OD pairs with small flows in the SC solution have zero flow in the MC solution; likewise, many OD pairs with small flows in the MC solution have zero flow in the SC solution.
Fig. 7. Comparison of solutions for all links without and with restrictions on truck use for single-class (SC) and multiple-class (MC) PAS-based traffic assignments of three car and truck matrices; total link flows shown on the left, link times shown on the right.

Fig. 7.1 SC total link flows without truck restrictions vs. MC total link flows with full restrictions: CS = .20

Fig. 7.2 SC total link flows without truck restrictions vs. MC total link flows with full restrictions: CS = .10

Fig. 7.3 SC total link flows without truck restrictions vs. MC total link flows with full restrictions: CS = .05

Fig. 7.4 SC link times without truck restrictions vs. MC link times with full restrictions: CS = .20

Fig. 7.5 SC link times without truck restrictions vs. MC link times with full restrictions: CS = .10

Fig. 7.6 SC link times without truck restrictions vs. MC link times with full restrictions: CS = .05
Fig. 8. North Avenue Select Link Analyses: Solutions without and with restrictions on truck use for single-class (SC) and multiple-class (MC) PAS-based assignments of three car-truck matrices; linear plots shown on the left, log plots on the right; both are needed for a full assessment.

Fig. 8.1 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .20

Fig. 8.2 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .10

Fig. 8.3 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .05

Fig. 8.4 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .20

Fig. 8.5 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .10

Fig. 8.6 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .05
Fig. 9. Harlem Avenue Select Link Analyses: Solutions without and with restrictions on truck use for single-class (SC) and multiple-class (MC) PAS-based assignments of three car-truck matrices; linear plots shown on the left, log plots on the right; both are needed for a full assessment.

Fig. 9.1 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .20

Fig. 9.2 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .10

Fig. 9.3 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .05

Fig. 9.4 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .20

Fig. 9.5 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .10

Fig. 9.6 SC total OD flows without truck restrictions vs. MC total OD flows with full restrictions: CS = .05
The following observations relate to the Harlem Avenue SB plots:

1. The total link flow is slightly higher for the MC solution for all three cost sensitivity values, and equal to or slightly greater than the link’s capacity;

2. The number of OD pairs using the link increases substantially as the CS value decreases;

3. The maximum link flow decreases from nearly 50 for CS = 0.20 to 10 for CS = 0.05;

4. Only in the solution for CS = 0.05 do substantial differences in OD flows occur for the SC and MC solutions;

5. The log plots reveal substantial differences regarding which OD pairs use this link for the SC and MC solutions, as indicated by the vertical and horizontal bars on all three plots.

In summary, the use of North Avenue EB by OD pairs is quite different in each cost sensitivity solution. Because each solution was well converged, and the proportionality conditions precisely applied, the observed differences are a result of the network restrictions and the use of multiple-class assignments, and not from imprecision or arbitrariness in the solutions.

5. Conclusions

Solutions of the single-class and multiple-class user-equilibrium traffic assignment problem were presented for three related trip matrices; the condition of proportionality was applied, so that the route flows and link flows for cars and trucks are uniquely determined. The Relative Gap of each assignment is less than 1E-10. These solutions were systematically compared for several selected links and pairs of alternative segments to identify the solution attributes and the effect of the various network representations.

The charts show that substantial differences occur with respect to total link flows by class in response to the different trip matrices and network representations. At the link flow level, multiple-class assignments are important to account for different network representations. Similar differences could occur if other generalized cost functions were assumed, such as with alternative values of time.

For the two selected links examined, substantial differences were also noted among the solutions for different trip matrices and network representations. Such differences depend substantially on the specific link selected, as illustrated by these two cases. Links on which trucks are fully restricted, of course, exhibit even larger differences; such links were examined, but are not shown here. The analysis of one pair of alternative segments showed relatively small differences in the proportion of OD flows using each segment by cars and trucks. These charts illustrate that the proportionality condition can be successfully applied to determine route flows uniquely. The application of this condition is considered to be a major advance in the quality of traffic assignments applied in transportation planning practice. In the future, we expect to continue our experiments to evaluate further the significance of this method.
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Appendix

The following formal description of the origin-destination-car and truck trip matrices used in the experiments reported in this paper is based on the Appendix in Bar-Gera and Boyce (2007).

Consider a study area divided into a set of zones $Z$, connected by transit services and by a road network. The road network consists of a set of nodes $N$, and a set of directional links $A$. In the Chicago regional model, there are 1790 zones, 12,982 nodes and 39,018 links. A route is a sequence of nodes $[v_1, v_2, \ldots, v_k]$ such that a given node pair corresponds to a link, $[v_j, v_{j+1}] \in A$. The set of available routes from origin $p \in Z$ to destination $q \in Z$ is $R_{pq}$, and the set of all routes is $R$.

The purpose of the model is to predict: (1) the origin-destination-mode (ODM) flow $d_{pq}^m$, in persons per hour, for every origin $p \in Z$, destination $q \in Z$, and mode $m \in \{c = \text{car}, t = \text{transit}\}$; (2) the distribution of car OD flows and the exogenously given truck OD flows to route flows $h_r$, for every route $r \in R$. Vehicle OD flows are the sum of OD person trips by car divided by a constant car occupancy factor ($cof = 1.2$ persons/car) and the truck OD flows $d_{pq}^{trk}$, in equivalent passenger cars per hour: $d_{pq}^{veh} = d_{pq}^c / cof + d_{pq}^{trk}$; hence $\sum_{r \in R_{pq}} h_r = d_{pq}^{veh}$. Total link flows are the result of route flow aggregation, $f_a = \sum_{r \in R} h_r \delta_{ar}^r$, where $\delta_{ar}^r = 1$, if link $a$ belongs to route $r$, and 0 otherwise. An ODM solution is feasible if it respects the constraints on total origin flows, $\sum_{mq} d_{pq}^m = d_{pq}^*$, and on total destination flows, $\sum_{pm} d_{pq}^m = d_{pq}^*$, where $d_{pq}^*$ is the given number of trips per hour departing from zone $p$, and, $d_{pq}^*$ is the given number of trips per hour arriving at zone $q$.

In the model implemented for the Chicago region, the total ODM flows amount to 1.513 million (1,513,211) persons per hour, while the total truck flows amount to 0.445 million (445,184) passenger-car-equivalents per hour, representative of the morning peak period (6:30 – 8:30 am) in 1990. The truck matrix was provided by the Chicago Area Transportation Study (CATS). The departing and arriving person flows by zone were factored from 24-hour totals also obtained from CATS.

The remaining model inputs refer to transit and road levels of service. Transit data are in-vehicle travel time in minutes, $c_{pq}^{ivt}$, out-of-vehicle travel time in minutes, $c_{pq}^{out}$, and fare in cents, $c_{pq}^{fare}$, for travel from origin $p \in Z$ to destination $q \in Z$ by transit. These values are fixed regardless
of flow. Origin-destination generalized cost by transit $u_{pq}^r$ is the weighted sum of the three components plus a constant bias; in these results, the transit bias is 0, and the weights are 0.25, 0.90, and 0.08, respectively.

Travel time $t_a$ on link $a$ is a function of total vehicle flow $t_a(f_a) = t_a^0 \left(1 + 0.15 \frac{f_a}{k_a} \right)$, where $t_a^0$ and $k_a$ are the free-flow travel time in minutes and the capacity of the link in vehicles per hour, respectively. The link generalized cost is $c_a(f_a) = t_a(f_a) + 0.15 \cdot l_a$, in minutes per vehicle, where $l_a$ is the link length in miles. The coefficient, 0.15 minutes/mile, reflects a combination of both the direct effect of distance on generalized cost and the indirect effect of fuel consumption. Fixed additional car costs $ac_{pq}^{car}$ account for parking fees and out-of-vehicle travel time at the origin and destination. The route generalized cost is defined as $c_r = \sum_{a \in A} t_a(f_a) \delta_r^a + ac_{pq}^{car}, r \in R_{pq}$.

The minimum OD generalized cost by road for each OD pair is $u_{pq}^{car} = \min_{r \in R_{pq}} \{c_r\}$. For every route $r \in R_{pq}$, define the excess cost as: $ec_r = c_r - u_{pq}^{car}$. The user-equilibrium assumption is that the excess cost of every used route is zero. Approximate UE solutions are evaluated by the maximum excess cost over all used routes. The total excess cost (TEC), also known as the Gap, is defined as $TEC = \sum_{r \in R} ec_r$. The total cost of road travel is $TC = \sum_{r \in R} c_r h_r = \sum_{a \in A} f_a c_a(f_a)$. The Relative Gap defined in terms of Total Cost is the ratio of the Total Excess Cost to Total Cost: $RG(TC) = TEC / TC$. Typically, the TEC, or Gap, is computed by performing an all-or-nothing (AON) assignment, given the link costs of solution $n$, $(f_a^n)$. Let $(f_a^{AON})$ be the link flows from that AON assignment. Then, $TEC = \sum_{a \in A} c_a(f_a^n)(f_a^n - f_a^{AON}) \geq 0$. For a definition of an alternative termination criterion based on the objective function of the user-equilibrium problem and its best lower bound, see Patriksson (1994, 96-97).

The ODM flows have the doubly-constrained logit form, $d_{pq}^m = R_p S_q \exp(-\beta u_{pq}^m)$, where $\beta$ is a cost sensitivity parameter, and $R_p, S_q$ are balancing factors ensuring that the constraints hold on total departing and arriving flows. Approximate solutions of this model are evaluated by the total misplaced ODM flow, $\sum_{p = q} \left| d_{pq}^m - R_p S_q \exp(-\beta u_{pq}^m) \right|$.

The combined model of user-equilibrium and origin-destination-mode choice with the specific structure described above can be formulated mathematically either as a fixed point problem, or as a convex optimization problem (Bar-Gera and Boyce, 2003). For the generalized link cost function stated above (separable, monotonically increasing), the equilibrium, or optimal, solution uniquely determines total link flows. Total link flows in turn uniquely determine link costs, route costs, and the set of minimum cost routes, referred to here as the set of UE routes. Of course, the route flows over this set of UE routes are not unique.
The model was solved by an origin-based assignment algorithm (OBA). Route flow solutions are described by a set of restricting a-cyclic subnetworks \( A_p \) for each origin, and origin-based approach proportions \( \alpha_{pa} \in [0,1] \), such that \( \sum_{a \in A_p} \alpha_{pa} = 1 \) for all \( p \in \mathbb{Z}, v \in N, v \neq p \), and \( \alpha_{pa} = 0 \) for all \( a \notin A_p \). The implicit set of routes from origin \( p \) is the set of all routes within \( A_p \), that is \( R_{pq}[A_p] = \{ r \in R_{pq} : a \subseteq r \Rightarrow a \in A_p \} \). The implicit route flows are given by \( h_r = \frac{d_{pq}^{\text{veh}}}{\alpha_{pa}} \prod_{a \subseteq r} \alpha_{pa} \). For the OBA algorithm, the number of UE routes per OD pair is unrelated to the number of iterations, unlike link-based algorithms.

The availability of route flows and approach proportions allows: (1) adjustments of origin-destination-mode flows while retaining the current route proportions, (2) adjustments of restricting subnetworks to accommodate more routes, and (3) efficient adjustments to approach proportions that utilize second order derivatives of the objective function. The resulting algorithm offers precise convergence for large-scale networks. For the specific model of the Chicago region presented here, the algorithm produced a solution with a maximum excess cost of 1E-13 in-vehicle minutes and a total misplaced ODM flow of 1E-10 person-trips per hour.

By itself, a precisely converged solution does not necessarily guarantee a set of routes that is similar to the true, unique set of UE routes. As link flows and link costs converge towards their equilibrium values, so should excess costs. Therefore, excess costs of UE routes should decrease continuously towards zero, while the excess cost of any non-UE route should converge to a strictly positive value. The minimum equilibrium excess cost of all non-UE routes, considered as the rejection gap, is strictly positive as well. In principle, if a threshold below the equilibrium rejection gap is chosen, then at a certain finite level of convergence the excess cost of all UE routes will be below the threshold, and the excess cost of all non-UE routes will be above the threshold.

We chose to include all routes with an excess cost below a threshold of 2E-12. The smallest excess cost of a rejected route is at least 30E-12, which determines the estimated rejection gap. Therefore, there are no routes with 2E-12 < excess cost < 30E-12. There are several reasons to believe that the chosen set of routes is probably similar to and perhaps identical with the true set of UE routes. One reason is the stability of the set of included routes in the final iterations of the origin-based assignment. Another reason is the order of magnitude difference between the chosen threshold and the estimated rejection gap. Additional reasons are discussed in Bar-Gera (2006), particularly the fact that the chosen set of routes maintains consistent consideration of alternative route segments, a fundamental property of sets of minimum cost routes in general and the set of UE routes in particular.
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