On Automobile Repairs and the Provision of Loaner Vehicles
by Dealers

by

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Abstract

In this note, we analyze two hitherto unstudied questions from the perspective of an automobile dealer that uses a particular temporal decision rule to provide loaner vehicles to customers who come to its facility with automobile repair requests. First, on the assumption that the basic criterion for the above mentioned decision rule is satisfied, we determine the expected repair times of automobiles for which our dealer provides customers with loaner vehicles. Second, we ascertain the average number of loaner vehicles that are out with customers.

Keywords: Automobile, Dealer, Loaner Vehicle, Repair, Uncertainty

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1. Introduction

Readers who either own or lease an automobile will recognize that like any other machine, an automobile will break down from time to time. When this happens, the owner and/or the lessor will need to bring his or her automobile to a repair shop which is, very frequently, the repair facility associated with the business operations of an automobile dealer.

When a customer brings his or her malfunctioning automobile to a dealer’s repair facility, this dealer will, under certain circumstances, provide the pertinent customer with a loaner vehicle. Now, the circumstances under which a dealer will provide repair seeking customers with loaner vehicles vary, but, generally speaking, the two key factors that are germane here are the cost to the dealer and the time it will take to repair the automobiles in question. For leased automobiles or for automobiles with unexpired warranties, the underlying automobile manufacturer and the dealer typically have existing cost sharing agreements in place and hence in these instances, from a dealer’s perspective, this cost issue is largely a deterministic matter.

In contrast, for automobiles that are no longer on a warranty of some sort, the dealer has considerable discretion in determining whether to provide customers with loaner vehicles at its, i.e., the dealer’s, expense. Here, the key variable is time. Put differently, customers with lengthy repair jobs tend to get loaner vehicles and customers with relatively short repair jobs do not get loaner vehicles. Instead, this latter group of customers is generally asked either to wait in the dealer’s premises or to arrange for its own transport to and from this dealer’s repair facility.

Readers with automobiles will agree that the above description of events is a fair characterization of automobile repairs and the provision of loaner vehicles by dealers in most settings in the United States of America. Despite the great practical relevance of the twin tasks of
repairing malfunctioning automobiles and providing loaner vehicles to customers, to the best of our knowledge, questions associated with these twin tasks have not been studied previously in either the economics or the regional science literatures. What has been studied thus far, but in isolation, are the two topics of automobile repairs and the behavior of automobile dealers in particular circumstances. Therefore, we now briefly survey this literature in the next two paragraphs and then we proceed to the two specific questions of this note.

Focusing first on automobile repairs, the impact that the regulation and the provision of consumer information have on automobile repairs has been analyzed by Webbink (1978). Kolodinsky (1995) argues that when it comes to automobile repairs, the traits of individuals are salient in explaining what she calls the “complaint behavior” of consumers. In their empirical analysis of warranty data, Majeske et al. (1997) seek to correct the biases that arise when a single individual contributes many automobile repairs to the repair population. Finally, in the context of automobile recalls, Rupp and Taylor (2002) have shown that the largest owner repair responses are associated with newsworthy hazardous defects in new domestic vehicles in their initial model year.

Focusing next on automobile dealers, Eckard (1985) has studied the effects that state level regulations on the entry of new automobile dealers have on the prices of new automobiles. Mathewson and Winter (1989) have analyzed the efficiency effects of laws that curb the ability of automobile manufacturers to either add new dealers in specific market areas or to terminate existing dealers. Between customers and dealers, who benefits most from automobile manufacturer incentive programs such as customer rebates and dealer discount options? Busse et al. (2006) show that customers benefit more from rebates but that dealers profit more from discount options. Finally, Arnold and Penard (2007) have examined the effects that dealer search costs and online
intermediaries have on the price setting process in the automobile market.

As noted above, given the absence of prior research on the related tasks of repairing malfunctioning automobiles and providing loaner vehicles to customers, in this note, we analyze two hitherto unstudied questions from the viewpoint of an automobile dealer that provides loaner vehicles using a particular temporal decision rule to customers who come to its facility with automobile repair requests. First, on the assumption that the primary criterion for the above decision rule is satisfied, we determine the expected repair times of automobiles for which our dealer provides loaner vehicles. Second, we ascertain the average number of loaner vehicles that are out with these customers. Before proceeding further, we would like to reiterate that this contribution of ours is a note and hence it is not our objective here to carry out an exhaustive analysis of the many facets of automobile repairs and the provision of loaner vehicles by dealers. This notwithstanding, in the concluding section 3, we do provide a reasonably detailed discussion of two ways in which the analysis contained in this note might be extended.

The rest of this note is organized as follows. First, in section 2.1, we describe a simple model of the interaction between customers who arrive at an automobile dealer’s facility with repair requests and, say, the dealer’s service manager who must decide which customers to give loaner vehicles to. Second, in section 2.2, we compute the mean repair time of automobiles for which the service manager provides loaner vehicles. Third, in section 2.3, we calculate the average number of loaner vehicles that are out with customers. Finally, section 3 concludes and then discusses two ways in which the research delineated in this note might be extended.

2. The Theoretical Framework

2.1. Preliminaries
Consider customers with malfunctioning automobiles who arrive at an automobile dealer’s repair facility in accordance with a stationary Poisson process with time independent rate $\lambda > 0$.\(^3\) We suppose that the repair times of the various malfunctioning automobiles are uniformly distributed on the interval $[a,b]$. The service manager begins the process of repairing malfunctioning automobiles immediately upon their arrival in the dealer’s repair facility.\(^4\) The exact time it will take to complete a particular repair job can be determined only upon the arrival of the relevant automobile.

Consistent with the discussion in section 1, we suppose that the service manager uses the following decision rule to allocate loaner vehicles to individual customers. If the repair time for a particular automobile takes longer than $\tau$ time units where $\tau \in [a,b]$, then the customer in question is provided with a loaner vehicle until his or her automobile repair job is completed. Given this description of our basic setup, our next task is to determine the expected repair time of automobiles for which our service manager will have to provide a loaner vehicle, given that the above temporal condition is satisfied.

2.2. The expected repair time

Recall that the arrival process of customers with malfunctioning automobiles is Poisson with rate $\lambda > 0$. Therefore, we deduce that the arrival process of customers who will require loaner vehicles is also Poisson but now with rate $\lambda p$ where $p = (b-\tau)/(b-a)$. Now, under the condition that

\(^3\) See Taylor and Karlin (1998, pp. 267-332) or Tijms (2003, pp. 1-32) for textbook treatments of the Poisson process.

\(^4\) We are implicitly assuming that there are ample repair facilities and that this is why the service manager is able to begin the process of repairing malfunctioning automobiles immediately upon their arrival in the dealer’s premises. If this were not the case then it is certainly likely that at least some arriving customers with malfunctioning vehicles would have to wait to receive repair service. It is possible to account for this last contingency by analyzing a “finite repair capacity” model. However, this would complicate our subsequent mathematical analysis without providing any fundamental new insight. Therefore, in the remainder of this paper, we suppose that the above stated assumption is valid.
the automobile of a customer requiring a loaner vehicle has a repair time \( R \) larger than \( \tau \), in symbols, we can write the conditional expectation we seek as \( E[R/R>\tau] \).

Our first task is to obtain a closed-form expression for the probability \( \text{Prob}(R>x/R>\tau) \) for any \( x\in[\tau,b) \). Now, from elementary properties of conditional probabilities and the fact that we are working with the uniform distribution, it follows that for any \( x\in[\tau,b) \), we can write

\[
\text{Prob}(R>x/R>\tau) = \frac{\text{Prob}(R>x)}{\text{Prob}(R>\tau)}.
\]

Using equation (1), we can simplify the conditional expectation \( E[R/R>\tau] \) that we seek. This simplification gives us

\[
E[R/R>\tau] = \frac{1}{\text{Prob}(R>\tau)} \int_{\tau}^{b} \frac{x}{b-a} \, dx = \frac{1}{p} \frac{1}{2} (\tau+b). \tag{2}
\]

The reader should note that instead of proceeding as we have above, we could follow a slightly different procedure to obtain the equation (2) expression for the conditional expectation \( E[R/R>\tau] \). In this slightly different procedure, we would first note that

\[
\text{Prob}(R/R>\tau) = \frac{\text{Prob}(R)}{\text{Prob}(R>\tau)} = \frac{1}{p} \frac{1}{b-a}. \tag{3}
\]

Then, using equation (3), we would infer that

\[
E[R/R>\tau] = \frac{1}{p} \int_{\tau}^{b} \frac{x}{b-a} \, dx = \frac{1}{p} \frac{(b^{2}-\tau^{2})}{2(b-a)} = \frac{1}{2} (\tau+b). \tag{4}
\]
It is clear that either of the two procedures delineated above gives us the same closed-form expression for the conditional expectation $E[R/R \geq \tau]$ that we seek.\(^5\)

The right-hand-side (RHS) of either equation (2) or (4) tells us that the expected repair times of automobiles for which our dealer’s service manager will have to provide a loaner vehicle, given that this manager’s decision rule is satisfied, is a simple additive function of the cutoff time $\tau$ and the right endpoint $b$ of the interval over which the uniformly distributed repair times are defined. Inspecting equation (2) or (4), it is straightforward to verify that, consistent with our intuition, when either the cutoff time $\tau$ or the right endpoint $b$ increases, the expected repair time under study also increases. We now proceed to the second and final task of this note and that is to ascertain the mean number of loaner vehicles that are out with customers.

**2.3. Mean number of loaner vehicles**

To obtain a closed-form expression for this average number, we shall apply the prominent $M/G/\infty$ model from queuing theory\(^6\) to our problem. This application tells us that the average number of loaner vehicles that are out with customers is given by the product of the rate of the arrival process of customers who will require loaner vehicles ($\lambda p$) and the expected repair time of automobiles for which our service manager will have to provide a loaner vehicle given in the RHS of either equation (2) or (4). In symbols, we get

$$
\text{Average Number of Loaner Vehicles} = \lambda p \times \frac{1}{2} (\tau + b) = \frac{\lambda (b^2 - \tau^2)}{2(b - a)}, \quad (5)
$$

\(^5\) We thank an anonymous referee for pointing out this slightly different procedure for deriving the closed-form expression for the conditional expectation $E[R/R \geq \tau]$.

Alternately, using equation (4), we get

\[
\text{Average Number of Loaner Vehicles} = \lambda p \times \frac{1}{p} \frac{b^2 - \tau^2}{2(b-a)} = \frac{\lambda (b^2 - \tau^2)}{2(b-a)}. \tag{6}
\]

Inspecting the RHS of either equation (5) or (6), it is straightforward to verify two comparative statics results that conform well with our intuition. First, as the rate \( \lambda \) of the Poisson arrival process of all customers goes up, the average number of loaner vehicles that are out with customers also goes up. Second and in contrast, as the cutoff time \( \tau \) for providing customers with loaner vehicles goes up, the mean number of loaner vehicles that are out with customers goes down. The answers provided to the two questions of this note in the equation pairs (2) and (4) and (5) and (6) have a significant bearing on the costs incurred by the automobile dealer under study. Hence, it is reasonable to suppose that an automobile dealer of the sort studied in this note will want to use these answers not only for planning purposes but also to aid the goal of conducting business operations so as to minimize the cost of such operations.

3. Conclusions

In this note, we analyzed two previously unstudied questions from the perspective of an automobile dealer that uses a particular temporal decision rule to provide loaner vehicles to customers who come to its facility with automobile repair requests. First, on the assumption that the basic criterion for the above decision rule is satisfied, we determined the expected repair time of automobiles for which our dealer would have to provide loaner vehicles. Second, we ascertained the average number of loaner vehicles that are out with customers.

The analysis in this note can be extended in a number of different directions. Here are two
suggestions for extending the research delineated here. First, it would be instructive to use the results obtained in this note to set up an optimization problem in which the cutoff time $\tau$ is determined *endogenously*. In this regard, one way to proceed is as follows. Following the suggestions made by an anonymous referee, let $c>0$ denote the time independent marginal (unit) unit cost accruing to our dealer’s service manager from providing a loaner vehicle. Similarly, let $f>0$ denote the time independent marginal (unit) cost accruing to an arriving customer if (s)he is *unable* to obtain a loaner vehicle. Then, from either equation (5) or (6) it follows that the expected cost to our dealer’s service manager from the provision of loaner vehicles is $\left[\frac{\lambda(b^2-\tau^2)}{2(b-a)}\right]c$. Some thought and the analysis leading up to equations (2) and (4) tell us that the expected cost to an arriving customer who is unable to obtain a loaner vehicle is $\lambda(1-p)E[R/R<\tau]$. One—but clearly not the only—way to determine the cutoff time endogenously would be to choose $\tau$ to minimize the (possibly weighted) sum of the two expected cost expressions that we have just identified.

Second, to model the fact that the automobile repair requests of arriving customers are more likely to occur in some time periods than in others, it would be useful to examine questions of the sort studied in this note with the proviso that the customers arrive at an automobile dealer’s repair facility in accordance with a non-stationary Poisson process. Studies of automobile repairs and the provision of loaner vehicles by dealers that incorporate these features of the problem into the analysis will provide additional insights into a set of issues that are both practically relevant and theoretically interesting.
References


