Urban unemployment, privatization policy, and a differentiated mixed oligopoly

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1. Introduction

This paper combines a traditional dualistic economy model with a differentiated mixed oligopoly model. In development economics, Harris and Todaro (1970), which is one of pioneer studies in dualistic economy, describes a dualistic economy by assuming the downward rigidity of wage in urban area and explains the occurrence of unemployment endogenously. Though there is no room for doubt that Harris and Todaro model is one of most important studies about dualistic economy analysis, it is hard to say that the setting necessarily accords with reality. Harris and Todaro model has been extended from the various viewpoints. Corden and Findlay (1975) extend Harris and Todaro model by taking account of mobile capital between regions.

As for a mixed oligopoly, De Fraja and Delbono (1990) construct a mixed oligopoly market, where private firms compete with a public firm in the homogeneous goods market. De Fraja and Delbono (1990) compares the mixed oligopoly, which a public firm is full nationalized, with the pure oligopoly, which a public firm is full privatization. Matsumura (1998) considers the model where the public firm maximizes the weighted average of social welfare and profit as the objective function and show that the partial privatization of public firm is optimal. Matsumura and Kanda (2005) take account of allow free entry of private firms in the mixed oligopoly.

Naito (2012) introduces a mixed duopoly into Fukuyama and Naito (2007) and analyzes the effect of privatization of public firm on urban employment. However, Naito (2012) deals with not differentiated goods but homogeneous goods. In the actual world, it is not natural to assume the homogeneous goods produced in urban area.

Fujiwara (2007) consider the quasi-linear utility function constructed by Ottaviano, Tabuchi, and Thisse (2002) and introduces product differentiation into a mixed oligopoly model. Though Fujiwara (2007) adopts the quasi-linear utility function of Ottaviano, Tabuchi, and Thisse (2002) in his paper, he does not take account of monopolistic competition. Thus, we combine a traditional dualistic economy model with a differentiated mixed oligopoly model to construct the model which enable to analyze the urban unemployment and the privatization of public firm of privatization under a differentiated mixed oligopoly. Thus, we combine a traditional dualistic economy model with a differentiated mixed oligopoly model to construct the model, which enables to analyze the urban unemployment and the privatization of public firm of privatization under a differentiated mixed oligopoly. We adopt Ottaviano, Tabuchi, and Thisse (2002) and Fujiwara (2007) to describe a product differentiation and a mixed oligopoly market. On the other hand, we use Harris and Todaro (1970) to describe a traditional dualistic economy. We consider the effect of privatization of public firm on urban unemployment or social welfare in this model.

The remainder of the paper is organized as follows. Section 2 presents the model and discusses the

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properties of market equilibrium. Based on section 2, section 3 clarifies the effects of public firm privatization on migration between urban area and rural area, urban unemployment, and social welfare. The last section presents concluding remarks.

2. The Model

We consider the economy consists of two regions in our model. One of them is urban area, the other is rural area. Following Harris and Todaro (1970), every household in the economy can migrate between regions without any cost. We consider the three types of household in this model. Indexes c, r, and u denote the household employed by the manufactured goods sector, those employed by agricultural goods sector, and those who are not employed in both sectors and reside in urban area, respectively. Now we consider two kinds of products: agricultural goods and manufactured goods. The manufactured goods and agricultural goods are produced in urban area and rural area, respectively. Though manufactured goods sector require labor and capital as input factors to produce their own products, the agricultural goods sector requires only labor as input factor. Though each manufactured goods are differentiated and are produced by one public firms and nprivate firms in urban area, the agricultural goods are homogeneous.

2.1 Household

We assume that the urban wage is fixed above the market-clearing level of labor market, and that it has downward rigidity attributable to the minimum wage system and so on. Particularly, let \( w_r \) represent the minimum wage in the urban area, which is higher than the rural wage \( w_r \). That rural wage is determined in the labor market of the agricultural goods sector and is equal to the marginal product of labor in the agricultural goods sector. Here we assume that households who reside in urban area and are not employed in manufactured goods sector do not have wage income, that is, \( w_u \) is equal to zero. Let \( L_c, L_r \) and \( L_u \) represent the number of each household type. Moreover, we normalize total population in the economy as one.

Since we assume that all households have common preference, they have the following same quasi-linear utility function as well as Ottaviano, Tabuchi, and Thisse (2002).

\[
U_l = \alpha \left( q_{l0} + \int_0^N q_{l(i)} di \right) - \frac{\beta}{2} \left( [q_{l0}]^2 + \int_0^N [q_{l(i)}]^2 di \right) - \frac{\gamma}{2} \left[ q_{l0} + \int_0^N q_{l(i)} di \right]^2 + z, \quad (l = c, r, u) \tag{1}
\]

where \( q_{l0}, q_{l(i)} \), and \( N \) denote the consumption of goods produced by public firm, that produced by i-th private firm, and agricultural goods, the number of private firm in manufactured goods sector, respectively. Since all households has one unit of labor and \( K \) units of capital, they allocate their wage income and \( rK \) to consumption of manufactured goods and agricultural goods. Thus, the budget constraint of each households are given by

\[
w_l + rK = p_0 q_{l0} + \int_0^N p(i) q_{l(i)} di + z \tag{2}
\]

Maximizing (1) subject to (2), demand functions of manufactured goods is as follows.

\[
q_{l0}^* = a - [b + c(N + 1)]p_0 + cP \tag{3}
\]

We assume that \( \alpha > 0 \) and \( \beta > \gamma > 0 \). As for detailed explanation of this utility function, see Ottaviano, Tabuchi, and Thisse (2002).
\[ q_i^e(i) = a - [b + c(N + 1)]p(i) + cP, \]  

where \( a, b, \) and \( c \) are \( \alpha / [\beta + \gamma N], \) \( 1 / [\beta + \gamma N], \) and \( \gamma / (\beta - \gamma)[\beta + \gamma N] \), respectively. Moreover, price index \( P \) is defined as follows: 
\[ P = p_0 + \int_0^N p(j)dj \]

Substituting (3) and (4) for (1), the indirect utility function is derived as follows.

\[
v_l = \frac{a^2(N+1)}{2b} - a \left( p_0 + \int_0^N p(i)d\tilde{i} \right) + \frac{b+c(N+1)}{2} \left( [p_0]^2 + \int_0^N [p(i)]^2 d\tilde{i} \right) - \frac{c}{2} \left[ p_0 + \int_0^N p(i)d\tilde{i} \right]^2 + w_l + r\tilde{k} \]

Though we assume that all households are mobile between regions, the wage in manufactured goods sector has downward rigidity attributable to the minimum wage system and so on and higher than the wage in agricultural goods sector. Thus, each household compares the expected utility in urban area with the utility in rural area. Now we define \( \lambda \) as the unemployment rate in urban area, that is,

\[
\lambda \equiv \frac{L_u}{L_c + L_u} \]

All households have no incentive to migrate between urban area and rural area when the expected utility in urban area is equal to the utility in rural area. Thus, migration equilibrium condition is given by the following equation.

\[
(1 - \lambda)v_c + \lambda v_u = v_r \quad \Leftrightarrow \quad (1 - \lambda)\tilde{w} = w_r \]

Since we assume that total population in the economy is normalized, the population constraint is as follows.

\[
L_c + L_u + L_r = 1 \]

Combining (6) with (8), the population constraint in the economy is revised as follows.

\[
L_c + (1 - \lambda)L_r = 1 - \lambda \]

### 2.2 Production

#### 2.2.1 Agricultural goods sector

We consider that two kinds of goods are produced in this economy. One of them is the agricultural goods and produced in rural area. On the other hand, the other is the manufactured goods and done in urban area. We assume that the market of agricultural goods is competitive. The agricultural goods sector has decreasing returns of scale with respect to labor. Particularly we specify the production function of agricultural goods sector as follows.

\[
Z = (L_r)^\sigma, \quad \sigma \in (0, 1) \]

The agricultural goods sector is competitive and the agricultural goods are homogeneous. Moreover, we assume that the wage has no downward rigidity in rural area. So the wage is equal to the marginal product in rural area.

\[
w_r = \sigma(L_r)^{\sigma-1} \]

#### 2.2.2 Manufactured goods sector

We assume that the manufactured goods sector is the differentiated mixed oligopoly. In the mixed oligopoly market, the public firm competes with private firms in the common market. Here we consider that the manufactured goods market is the monopolistic competition as well as Ottaviano, Tabuchi, and Thisse
(2002). Supposed that the number of private firm is so large, the behavior of firms in urban area does not affect the price index in the manufactured goods market regardless of public firm or private firms. First of all, we derive the total demand of each manufactured goods, which is denoted by $Q_0$, $Q(i)$. Since the demand for each manufactured goods is independent of household’s income due to quasi-linear function, each total demand function of manufactured goods sector is given by

\[ Q_0 = a - [b + c(N + 1)]p_0 + cF \]  
\[ Q(i) = a - [b + c(N + 1)]p(i) + cP, \quad i \in [0, N] \]  

Here we assume that $Q_j$ units of production require $L_{jc}/m$ units labor and one unit of capital following Naito (2012). Though the private firms pursue their profit in mixed oligopoly, the public firm determines his production to maximize the social welfare. Thus, we define the social welfare function to analyze the behavior of public firm. Let $W$ represent the social welfare in this model as follows.

\[ W = \frac{a^2(N+1)}{2b} - a \left( p_0 + \int_0^N p(i)di \right) + \frac{b+c(N+1)}{2} \left( [p_0]^2 + \int_0^N [p(i)]^2di \right) \]
\[ - \frac{c}{2} \left[ p_0 + \int_0^N p(i)di \right]^2 + \pi_0 + \int_0^N \pi(i)di + \bar{w}(L_0^0 + \int_0^N L_c^i di) + \pi_r + w_rL_r + r(N + 1) \]  

where $\pi_r$ is the profit of rural sector. Since we assume that the number of private firm is large as well as Ottaviano, Tabuchi, and Thisse (2002), both public firm and private firms do not affect the price index of manufactured goods market. Here we consider that the government owns the share of $(1 - \theta)$ of public firm as well as Matsumura (1998). Thus, the purpose of public firm (firm 0) is to maximize the weighted average of social welfare and its profit, which is defined by $V(\theta)$.

\[ V(\theta) = \pi_0 + (1 - \theta)W \]  

Since each private firms determine the price to maximize their own profit function, the first order condition for profit maximization is given by

\[ \frac{\partial \pi(i)}{\partial p_0} = a - 2[b + c(N + 1)]p_i + cP + [b + c(N + 1)]m\bar{w} = 0 \]  

On the other hand, since the public firm maximizes the weighted average of social welfare and its profit, the first order condition is as follows.

\[ \frac{\partial V(\theta)}{\partial p_0} = a - (1 + \theta)[b + c(N + 1)]p_0 + cP + \theta m\bar{w}[b + c(N + 1)] = 0 \]  

Solving (16) and (17), we derive the following equilibrium prices of each goods.\(^3\)

\[ p_0^* = \frac{a}{(1 + \theta)[b + c(N + 1)]} \left( \frac{(\theta + 1)^2c + N^2}{2(1 + \theta)b + (2 + (1 + \theta)N)c} + \frac{2\theta[1 + \theta]|b|(1 + \theta)(N + 1)c}{2(1 + \theta)^2b + (2 + (1 + \theta)N)c} \right) m\bar{w} \]  
\[ p^* = \frac{(1 + \theta)a}{2(1 + \theta)b + [2 + (1 + \theta)N]c} + \frac{(1 + \theta)b(N + 1)c}{2(1 + \theta)b + [2 + (1 + \theta)N]c} m\bar{w} \]  

Moreover, substituting (18) and (19) for price index $P$, we derive the equilibrium price index as follows.

\[ P^* = \left( \frac{2(\theta + 1 + \theta)N}{2(1 + \theta)(b + (2 + (1 + \theta)N)c)} a + \frac{[b + c(N + 1)]c(2 + N)\theta + N}{2(1 + \theta)b + (2 + (1 + \theta)N)c} \right) m\bar{w} \]  

Finally, we refer to the capital market. Since every firm requires one unite of capital as a fixed input for

\(^3\) Here we assume that each private firm are symmetric: $p = p(i) \in (0, N)$
production. Total number of manufactured goods sector is equal to \( N+1 \). On the other hand, every household in the economy has \( \bar{k} \) units of capital as initial endowment and total number of households is one. Thus, the following equation must hold due to market clear condition in equilibrium.\(^4\)

\[
N + 1 = \bar{k}
\]

(20)

3. Migration between urban area and rural area

In the previous section, we analyze the behavior of households and firms in the model. Using equilibrium derived in previous section, we consider the effect of privatization of public firm on unemployment and social welfare. Taking account of (12), (13), (18), (19), the definition of price index manufactured goods, and production function, the labor demand of manufactured goods sector \( \bar{L} \) is as follows.

\[
\bar{L}_c = \left[ N + 1 - b \left( \frac{2^\theta + (1+\theta)N}{2(1+\theta)N} \right) \right] am - b \left( \frac{b+c(N+1)}{2(1+\theta)N} \right) m^2 \bar{w} - \left( \frac{1}{2(1+\theta)b+(2(1+\theta)N)c} \right) m^2 \bar{w}
\]

(21)

Combining (7) with (11), we can derive the following equation.

\[
L_r = (1 - \lambda)^{\frac{1}{\sigma}} \left( \frac{\bar{w}}{\sigma} \right)^{\frac{1}{\sigma}}
\]

(22)

Substituting (21) and (22) for (9), the equilibrium unemployment in urban area is determined by the following equation.

\[
\left[ N + 1 - b \left( \frac{2^\theta + (1+\theta)N}{2(1+\theta)N} \right) \right] am - b \left( \frac{b+c(N+1)}{2(1+\theta)N} \right) m^2 \bar{w} + (1 - \lambda)^{\frac{1}{\sigma}} \left( \frac{\bar{w}}{\sigma} \right)^{\frac{1}{\sigma}} = 1 - \lambda
\]

(23)

Applying the implicit function theorem to (23) to analyze the effect of privatization of public firm on urban unemployment, \( d\lambda/d\theta \) is derived as follows.

\[
\frac{d\lambda}{d\theta} = - \left( \frac{1}{(\frac{\bar{w}}{\sigma})(\frac{1+\theta)}{\sigma^{\sigma+1}}) \right) \left\{ \frac{\bar{w}}{\sigma} \right\} \left\{ \frac{\bar{w}}{\sigma} \right\} \left\{ \frac{\bar{w}}{\sigma} \right\} > 0
\]

(24)

Since both denominator and numerator in (24) are positive, the sign of (24) is determined uniquely. So we derive the following proposition.

**Proposition 1**

The progress of public firm’s privatization in urban area leads to deteriorate urban unemployment.

Next we consider the effect of public firm’s privatization on social welfare in equilibrium. Let \( \lambda^* \) represent \( \lambda \) to hold (23). Substituting \( \lambda^* \) for (22), the equilibrium rural employment are given by

\[
L_r^* = (1 - \lambda^*)^{\frac{1}{\sigma}} \left( \frac{\bar{w}}{\sigma} \right)^{\frac{1}{\sigma}}
\]

(25)

Since we define (14) as social welfare function, we substitute (18), (19), equilibrium price index, and (25) for (14) to derive the equilibrium social welfare function. Let \( W^* \) represent the equilibrium social welfare function. Thus, the equilibrium social welfare function is as follows.

\[
W^* = \frac{\sigma^2(N+1)}{2\sigma} - \frac{b+c(N+1)}{2} \left( \left[ P_0 \right]^2 + N[P^*]^2 \right) + \frac{2}{\sigma} [P^*]^2 + [L_r^*]^\sigma
\]

(26)

\(^4\) Thus, the number of private firms in manufactured goods sector is determined by the household’s initial endowment of capital.
Differentiating (26) with respect to $\theta$, the effect of public firm’s privatization on social welfare in equilibrium is as follows.

$$\frac{\partial W^*}{\partial \theta} = -[b + c(N + 1)] \left( P_0 \frac{\partial p^0}{\partial \theta} + Np^* \frac{\partial p^*}{\partial \theta} \right) + cP^* \frac{\partial P^*}{\partial \theta} + \frac{\partial}{\partial \theta} \left( (L_r^*)^{\sigma} \right)$$  \(27\)

Here we define the first term, the second term, and third term as price effect, price index effect, and rural employment effect, respectively. Generally, the sign of (27) is not determined uniquely. Though we know that the effect of privatization of public firm on price index effect and rural employment effect are positive with easy calculation, the effect of privatization of public firm on price index is ambiguous. If price index effect and the price effect are superior to rural employment effect, the progress in privatization of public firm leads to improve social welfare. Thus, we derive the following proposition.

**Proposition 2**

*When the price effect and the rural employment effect, are superior (inferior) to price index effect, the progress in privatization of public firm leads to improve (make worse) social welfare.*

### 4. Concluding remarks

We combine the traditional Harris and Todaro model with mixed oligopoly and introduce monopolistic competition into this model. As the results of our analysis, we show that the progress in privatization of public firm make worse urban unemployment. Moreover, the privatization can improve social welfare if the price effect and rural employment effect is large relatively. For simplification, we do not refer to capital market. Thus, this point is our subject in future research.

**References**


