

## Sustainable Growth of Forest

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### Abstract

In our modern economy, we are faced with serious environmental problems suffering from pollution the firms and households generate in the process producing or consuming. The forest stock in many countries contributes is functioning to purify pollution. However, individuals usually don't cultivate since forest is public goods in nature. Also forest stock are used inputs in the production of goods. The forest stock is dying in many countries. Environment becomes worse rapidly in the world. The government must keep the clean environment of economy to complete the sustainable growth.

In this paper, we consider the economy in which pollution is occurring in the production process where the consumption goods are produced using the forest resources as inputs. The government cultivates to deserve the environment for next generation. We suppose that the government imposes taxes to finance the cultivating costs for environmental preservation.

We derive the optimal condition for investment for capital and investment for cultivating forest for future generation in the steady state economy. We analyze whether the government policy to invest for cultivation forest financing by income tax and consumption tax can attain the efficient state.

We conclude that in the case where the government uses income tax, the income level is lower than the income of the social optimal case. The steady state pollution level in this case is also lower than the social optimal case, therefore the optimal forest stock level is also lower than the optimal.

**Key Words:** Forest Preservation Tax, Optimal Sustainable Growth of Forest,

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### 1. Introduction

In our modern economy, we face serious environmental problems due to the pollution the firms and households produce in the production and consumption process. The forest stock in many countries is functioning to purify. However, individuals usually do not cultivate since the forest stock is public goods in nature. The forest stock is dying in many countries, and the world environment is rapidly deteriorating. The government must produce a policy to keep the clean environment to attain the sustainable growth of economy (Pittel(2002)).

In this paper, we consider the economy in which one generation produces goods and pollution in the production process. Then the government plans to invest for cultivating the forest to preserve the environment for the next generation. We suppose that the government imposes taxes to finance the cultivating costs for environmental preservation.

We derive the optimal condition for investment of capital and cultivating forest for future generations in the steady state economy. We analyze whether or not the government policy to invest for cultivation forest, financing by income tax and consumption tax, can attain the efficiency.

We conclude that in the case where the government uses income tax, the government can attain the optimal state, but the government uses consumption tax, the government cannot attain the optimal. In the latter case, the realized income level is lower than the level of income in the social optimal case. The steady state pollution level in this case is lower than the social optimal one.

### 2. Model

We consider an economy in which individuals live for one period. We call individuals who live in  $t$  period  $t$  generation. The number of individuals in each period is constant and normalized one. They produce goods using capital stock. Individuals allocate the produced goods to their consumption, investment for capital stock and forest cultivation.

We assume that the production process generates pollution, and the forest stock can purify the pollution.

We suppose that each individual of all generation care for the next generation and that each individual of  $t$  generation has the following utility function,

$$u(c_t, Z_t) + (1 + \delta)^{-1}u(c_{t+1}, Z_{t+1}) \quad (1)$$

where,  $c_t$  and  $c_{t+1}$  are the amounts of consumption of generation  $t$  and  $t + 1$  in the period  $t$  and  $t + 1$  respectively. We assume that utility function is invariant over generations.

We define the following variables  $y_t, I_t, I_t^M, K_t, M_t, Z_t, \delta, \rho, \zeta$  as products, investment, forest conservation investment, capital stock, forest stock, in the period  $t$ , social discount rate, the rate of depreciation in capital stock, the rate of depreciation in forest stock.  $\alpha, \beta, m$  define as the rate of produced pollution by the capital stock used in production process, the purification rate of pollution by the forest stock, the rate of the forest increase by cultivation, respectively.

The following relations must be satisfied in any period.

$$c_t = y_t - I_t - I_t^M \quad (2)$$

$$y_t = f(K_t) \quad (3)$$

$$K_{t+1} = (1 - \rho)K_t + I_t \quad (4)$$

$$M_{t+1} = (1 - \zeta)M_t + mI_t^M \quad (5)$$

$$Z_t = \alpha K_t - \beta M_t \quad (6)$$

### 3. The optimization problem

We define the social welfare function as

$$W = \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau}) (1 + \delta)^{-(t+\tau)} \quad (7)$$

where,  $\delta$  is the social discount rate which is assumed to be equal to individual's one. Then the optimal problem is

$$\max_{I_t, I_t^M} \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau}) (1 + \delta)^{-(t+\tau)} \quad (8-1)$$

with subject to (2) -(6).

We define the state evaluation function,

$$J(K_t, M_t) = \max_{I_t, I_t^M} \{u(c_t, Z_t) + (1 + \delta)^{-1}J(K_{t+1}, M_{t+1})\} \quad (9-1)$$

We restrict our attention to a steady state path, assuming that an optimal choice exists and converges to a steady state. Then, we obtain

$$J_K = mJ_M \quad (10)$$

Furthermore, on the steady state path, the following relations must be hold

$$J_K = u_c f'(K) + u_Z \alpha + (1 + \delta)^{-1}J_K(1 - \rho) \quad (11)$$

$$J_M = -u_Z \beta + (1 + \delta)^{-1}J_M(1 - \zeta) \quad (12)$$

Using (10), we have the optimal condition in steady state,

$$\frac{u_c f'(K) + u_Z \alpha}{\delta + \rho} = \frac{-u_Z m \beta}{\delta + \zeta} \quad (13)$$

The left hand side of (13) is the social marginal benefits of one unit of the investments in capital stock, and the right hand side of (13) is the one of the unit of the investment in the forest stock.

#### 4. Individual behavior and the government: the income tax case

In this section, we assume that the government imposes the income tax  $T_t$  to finance the cost of forest cultivation investment and that the government determines tax rate so as to maximize individual welfare subjected to individual behavior and the government budget equation.

$$T_t y_t = I_t^M \quad (14)$$

We assume that the utility of individual is the sum of his own utility and the discounted utility of his child. Moreover, that the individual chooses his own consumption and investment for the capital of his child to maximize his utility, subject to his budget and the existing stock of inherited capital and forest by his parent.

Then his maximization problem of generation  $t$  is formalized as,

$$\max_{I_t} \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau})(1 + \delta)^{-(t+\tau)} \quad (8-2)$$

subject to

$$c_t = (1 - T_t)y_t - I_t \quad (2')$$

$$y_t = f(K_t) \quad (3)$$

$$K_{t+1} = (1 - \rho)K_t + I_t \quad (4)$$

$$M_{t+1} = (1 - \zeta)M_t + mT_t y_t \quad (5')$$

$$Z_t = \alpha K_t - \beta M_t \quad (6)$$

We define the evaluate function as

$$J(K_t, M_t) = \max_{I_t} \{ u(c_t, Z_t) + (1 + \delta)^{-1}J(K_{t+1}, M_{t+1}) \} \quad (9-2)$$

Then, optimal condition of individual restricting the case of the steady state

$$u_c = (1 + \delta)^{-1}J_K \quad (15)$$

Individual invest so as to satisfy (15). The Government chooses the tax rate  $T$  to maximize the social welfare (7). The state evaluate function of government in this case is

$$J(K_t, M_t) = \max_{T_t} u(c_t, Z_t) + (1 + \delta)^{-1}J(K_{t+1}, M_{t+1}) \quad (9-3)$$

The first conditions are on the steady state path are

$$J_K = (1 + \delta)u_c \quad (16)$$

$$mJ_M = (1 + \delta)u_c \quad (17)$$

From (16) and (17), we obtain

$$J_K = mJ_M \quad (10)$$

Furthermore, on the steady state path, the following condition must be hold

$$J_K = u_c f'(K) + u_Z \alpha + (1 + \delta)^{-1} J_K (1 - \rho) \quad (11)$$

$$J_M = -u_Z \beta + (1 + \delta)^{-1} J_M (1 - \zeta) \quad (12)$$

Therefore, we conclude that the government tax policy, which levies on income tax to finance cultivating, can attain the efficiency in the steady state.

### 5. The consumption tax case

In this section, we assume that the government imposes the consumption tax to finance the cost of forest cultivation Investment. The government determines the consumption tax rate to maximize individual welfare subjected to individual behavior and the government budget equation.

$$(T_t - 1)c_t = I_t^M \quad (18)$$

The maximization problem of generation  $t$  in this case is

$$\max_{I_t} \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau}) (1 + \delta)^{-(t+\tau)} \quad (8-3)$$

with subjected to

$$T_t c_t = y_t - I_t \quad (2'')$$

$$y_t = f(K_t) \quad (3)$$

$$K_{t+1} = (1 - \rho)K_t + I_t \quad (4)$$

$$M_{t+1} = (1 - \zeta)M_t + m(T_t - 1)c_t \quad (5'')$$

The first order conditions of individual maximization problem on the steady state path is

$$-u_c/T + (1 + \delta)^{-1} J_K = 0 \quad (19)$$

The government determines the tax rate to maximize the social welfare function with individual behavior given. The maximization problem of the government is

$$\max_{T_t} \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau})(1 + \delta)^{-(t+\tau)} \quad (8-4)$$

with subjected to budget constraint (18) and individual behavior (19).

We have the following three optimal conditions by the same procedure as the previous section.

$$J_K = mJ_M \quad (10)$$

$$J_K = u_c f'(K) + u_z \alpha + (1 + \delta)^{-1} J_K (1 - \rho) \quad (11)$$

$$J_M = -u_z \beta + (1 + \delta)^{-1} J_M (1 - \zeta) \quad (12)$$

On the steady state, using (10)-(12), we have the optimal condition in steady state in this case,

$$\frac{u_c f'(K)/T + u_z \alpha}{\delta + \rho} = \frac{-u_z m \beta}{\delta + \zeta} \quad (20)$$

Since  $T$  is larger than 1, in the case of the government use consumption tax to finance cultivating cost, the left hand side of (20) is less than the right hand side. Therefore, in the consumption tax case, the income level is lower than the income of the social optimal case. The steady state pollution level in this case is lower than the social optimal case therefore the optimal forest stock level is also lower than the optimal case.

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